**EECE 210**

**Final Exam, December 13, 2014**

3%

**A.** Two coils are wound on a high-permeability toroidal core of 2 cm2 cross-sectional area, with coil 2 open-circuited. If the magnetic flux density *B* in the core, in the clockwise sense, due to current in coil 1, is ***decreasing*** at a constant rate of 0.1 Wb/cm2/s, determine the magnitude and sign of *v*2, assuming coil 2 has 10 turns.

**Solution:** *λ*21 = *BAN*2; . If *B* is decreasing with time, and current is allowed to flow in coil 2 by connecting a resistor across coil 2 terminals, current will have to enter at the upper terminal of coil 2 to produce a flux aiding that in the core. It follows that the sign of *v*2 is negative.

**Version 1:** *N*2 = 10 turns, *v*2 = -0.2*N*2 = -2 V

**Version 2:** *N*2 = 15 turns, *v*2 = -0.2*N*2 = -3 V

**Version 3:** *N*2 = 20 turns, *v*2 = -0.2*N*2 = -4 V

**Version 4:** *N*2 = 25 turns, *v*2 = -0.2*N*2 = -5 V

**Version 5:** *N*2 = 30 turns, *v*2 = -0.2*N*2 = -6 V

3%

**B.** The switch is closed at *t* = 0. Determine *vC*(*t*), *t* ≥ 0+, assuming an initial value *VC*0 = 5 V and *VSRC* = 10 V.

**Solution:** *VCF* = *VSRC*, *τ* = 2 s, so that  V, *t* ≥ 0+, *t* is in s.

**Version 1:** *VSRC* = 10 V, V, *t* ≥ 0+, *t* is in s

**Version 2:** *VSRC* = 15 V, V, *t* ≥ 0+, *t* is in s

**Version 3:** *VSRC* = 20 V, V, *t* ≥ 0+, *t* is in s

**Version 4:** *VSRC* = 25 V, V, *t* ≥ 0+, *t* is in s

**Version 5:** *VSRC* = 30 V, V, *t* ≥ 0+, *t* is in s

3%

**C.** The switch is closed at *t* = 0, with an initial voltage *VC*0 = 5 V, and no energy stored in the inductor. Determine *v*(0+), *iL*(0+), and *iC*(0+), assuming *R* = 2 Ω.

**Solution:** *v*(0+) = *Vc*0, *iL*(0+) = 0, *iC*(0+) = -*Vc*0/*R*.

**Version 1:** *VC*0 = 5 V; *v*(0+) = 5 V, *iL*(0+) = 0, *iC*(0+) = -*Vc*0/*R* = -5/2 = -2.5 A

**Version 2:** *VC*0 = 10 V; *v*(0+) = 10 V, *iL*(0+) = 0, *iC*(0+) = -*Vc*0/*R* = -10/2 = -5 A

**Version 3:** *VC*0 = 15 V; *v*(0+) = 15 V, *iL*(0+) = 0, *iC*(0+) = -*Vc*0/*R* = -15/2 = -7.5 A

**Version 4:** *VC*0 = 20 V; *v*(0+) = 20 V, *iL*(0+) = 0, *iC*(0+) = -*Vc*0/*R* = -20/2 = -10 A

**Version 5:** *VC*0 = 25 V; *v*(0+) = 25 V, *iL*(0+) = 0, *iC*(0+) = -*Vc*0/*R* = -25/2 = -12.5 A

3%

**D.** Specify whether the response for *t* ≥ 0+ in the preceding problem is overdamped, critically damped, or underdamped, assuming *R* = 0.2 Ω.

**Solution:**  rad/s; 

**Version 1:** *R* = 0.2 Ω; *αp* = 50/0.2= 250 rad/s, response is overdamped

**Version 2:** *R* = 0.4 Ω; *αp* = 50/0.4= 125 rad/s, response is overdamped

**Version 3:** *R* = 0.5 Ω; *αp* = 50/0.5= 100 rad/s, response is critically damped

**Version 4:** *R* = 0.8 Ω; *αp* = 50/0.8= 62.5 rad/s, response is underdamped

**Version 5:** *R* = 1 Ω; *αp* = 50/1= 50 rad/s, response is underdamped

3%

**1.** Determine *Va* assuming all resistances are 1 Ω and the current sources are 1 A each.

**Solution:** From symmetry, each half circuit is as shown. The current in the 2 Ω resistor is *I*/4 A, and *Va* = *I*/2 A.

**Version 1:** *I* = 1 A, *Va* = 0.5 V

**Version 2:** *I* = 2 A, *Va* = 1 V

**Version 3:** *I* = 3 A, *Va* = 1.5 V

**Version 4:** *I* = 4 A, *Va* = 2 V

**Version 5:** *I* = 5 A, *Va* = 2.5 V

3%

**2.** Determine Norton’s current **IN** and Norton’s admittance *YN*, looking into terminals ‘ab’, assuming **VSRC** = 5∠90° V.

**Solution:** When terminals ‘ab’ are short-circuited, **IN** = **IX** = **VSRC**/*j*5 A. When terminals ‘ab’ are open circuited, **VTH** = -*j*0.5**IX** = -*j*0.5**VSRC**/*j*5; when a test source is applied, with **VSRC** = 0, then **IX** = 0, so that the independent source behaves as an open circuit, and *YN* = 1/(-*j*5) = 2 S.

**Version 1:** **VSRC** = 5∠90° V, **IN** = **VSRC**/*j*5 = 1∠0°A, *YN* = *j*2 S

**Version 2:** **VSRC** = 10∠90° V, **IN** = **VSRC**/*j*5 = 2∠0°A, *YN* = *j*2 S

**Version 3:** **VSRC** = 15∠90° V, **IN** = **VSRC**/*j*5 = 3∠0°A, *YN* = *j*2 S

**Version 4:** **VSRC** = 20∠90° V, **IN** = **VSRC**/*j*5 = 4∠0°A, *YN* = *j*2 S

**Version 5:** **VSRC** = 25∠90° V, **IN** = **VSRC**/*j*5 = 5∠0°A, *YN* = *j*2 S

3%

**3.** Determine the net average power delivered in the circuit, assuming **VSRC** = 10∠0° V.

**Solution:** The net average power delivered is equal to the average power dissipated in the 4 Ω resistor. When the current source is set to zero, the current in the inductive branch is  A. When the voltage source is set to zero, the current in the inductive branch is  A. By superposition, the total current in the inductive branch is . The magnitude of this current is , and the average power dissipated is  W.

**Version 1:** **VSRC** = 10∠0° V, = 2 W

**Version 2:** **VSRC** = 15∠0° V, = 8 W

**Version 3:** **VSRC** = 20∠0° V, = 18 W

**Version 4:** **VSRC** = 25∠0° V, = 32 W

**Version 5:** **VSRC** = 30∠0° V, = 50 W

3%

**4.** Determine **I** so that **Vo** = 0, assuming that *X* = 2 Ω.

**Solution:** When **Vo** = 0, the circuit becomes as shown, with **Va** = **Vb**. It follows that *j*4 = *j*X**I**, so that **I** = (4/*X*)∠0° A.

**Version 1:** *X* = 1 Ω, **I** = 4/*X =* 4∠0°A

**Version 2:** *X* = 2 Ω, **I** = 4/*X =* 2∠0°A

**Version 3:** *X* = 4 Ω, **I** = 4/*X =* 1∠0°A

**Version 4:** *X* = 8 Ω, **I** = 4/*X =* 0.5∠0°A

**Version 5:** *X* = 10 Ω, **I** = 4/*X =* 0.4∠0°A

3%

**5.** Determine *iS*(*t*), assuming *vSRC*(*t*) = 4cos100*t* V.

**Solution:** *jωL* = *j*100×10-2 = *j* Ω; -*j*/*ωC* = -*j*/(100×10-2) = -*j*. It follows that the impedance of the paralleled *RLC* elements is 2 Ω; *iS*(*t*) = *vSRC*(*t*)/2 = 2cos100*t* A

**Version 1:** *vSRC*(*t*) = 4cos100*t* V; *iS*(*t*) = *vSRC*(*t*)/2 = 2cos100*t* A

**Version 2:** *vSRC*(*t*) = 6cos100*t* V; *iS*(*t*) = *vSRC*(*t*)/2 = 3cos100*t* A

**Version 3:** *vSRC*(*t*) = 8cos100*t* V; *iS*(*t*) = *vSRC*(*t*)/2 = 4cos100*t* A

**Version 4:** *vSRC*(*t*) = 10cos100*t* V; *iS*(*t*) = *vSRC*(*t*)/2 = 5cos100*t* A

**Version 5:** *vSRC*(*t*) = 12cos100*t* V; *iS*(*t*) = *vSRC*(*t*)/2 = 6cos100*t* A

3%

**6.** Determine the maximum magnitude of *vL* and its phase angle at this magnitude, given *v****SRC*** = 2cos*ωt* V, and *ω* assuming any value between 0 and ∞.

**Solution:** Maximum current occurs at rad/s and is  A. It follows that  =  V.

**Version 1:** *v****SRC*** = 2cos*ωt*, |*vL*|max = 2 V, *θL* = 90°

**Version 2:** *v****SRC*** = 4cos*ωt*, |*vL*|max = 4 V, *θL* = 90°

**Version 3:** *v****SRC*** = 6cos*ωt*, |*vL*|max = 6 V, *θL* = 90°

**Version 4:** *v****SRC*** = 8cos*ωt*, |*vL*|max = 8 V, *θL* = 90°

**Version 5:** *v****SRC*** = 10cos*ωt*, |*vL*|max = 10 V, *θL* = 90°

3%

**7.** Determine *Zin*, assuming *ω* = 10 rad/s and *C* = 20 mF.

**Solution:** *Leq* for the two coupled coils is (2 + 2 – 2) = 2 H; in parallel with 2 H, this gives 1 H, or *j*10 Ω. The two capacitors in series are C/2, having an impedance -*j*0.2/*C* Ω, when *C* is in F. It follows that *Zin* = *j*10 – *j*0.2/*C* Ω.

**Version 1:** *C* = 20 mF, *Zin* = *j*10 – *j*0.2/0.02 *= j*10 – *j*10 *=* 0

**Version 2:** *C* = 10 mF, *Zin* = *j*10 – *j*0.2/0.01 *= j*10 – *j*20 *=-j*10 Ω

**Version 3:** *C* = 5 mF, *Zin* = *j*10 – *j*0.2/0.005 *= j*10 – *j*40 *=-j*30 Ω

**Version 4:** *C* = 4 mF, *Zin* = *j*10 – *j*0.2/0.004 *= j*10 – *j*50 *=-j*40 Ω

**Version 5:** *C* = 2 mF, *Zin* = *j*10 – *j*0.2/0.002 *= j*10 – *j*100 *=-j*90 Ω

3%

**8.** Determine *Zin*, assuming *X* = 2 Ω.

**Solution:** With a test source applied, the voltage across the autotransformer is **V1** – **V1** = 0. It follows that **VT**/**IT** = *Zin* = -*jX* Ω.

**Version 1:** *X* = 2 Ω, *Zin* = -*jX = -j*2Ω

**Version 2:** *X* = 4 Ω, *Zin* = -*jX = -j*4Ω

**Version 3:** *X* = 6 Ω, *Zin* = -*jX = -j*6Ω

**Version 4:** *X* = 8 Ω, *Zin* = -*jX = -j*8Ω

**Version 5:** *X* = 10 Ω, *Zin* = -*jX = -j*10Ω

3%

**9.** Determine the total energy delivered by the battery from *t* = 0, when the switch is closed, till *t* → ∞, assuming *VSRC* = 10 V and no initial charge on the capacitor.

**Solution:** The current is , *W* == = . Alternatively, the integral of current is the final charge *q* on the capacitor, which equals *CVSRC*. The total energy delivered by the battery is *W* = *qVSRC* = .

**Version 1:** *VSRC* = 10 V, *W* = 10-2×100 = 1 J

**Version 2:** *VSRC* = 11 V, *W* = 10-2×121 = 1.21 J

**Version 3:** *VSRC* = 12 V, *W* = 10-2×144 = 1.44 J

**Version 4:** *VSRC* = 13 V, *W* = 10-2×169 = 1.69 J

**Version 5:** *VSRC* = 14 V, *W* = 10-2×196 = 1.96 J

3%

**10.** The switch is opened at *t* = 0, with no energy initially stored in the circuit. Determine *vS*(*t*),

*t* ≥ 0+, assuming *ISRC* = 1 A dc.

**Solution:** , ,

*vS*(*t*) = *vC*(*t*) + *vL*(*t*) = 5*ISRC*.

**Version 1:** *ISRC* = 1 A, *vS*(*t*) = 5*ISRC* = 5 V

**Version 2:** *ISRC* = 1.5 A, *vS*(*t*) = 5*ISRC* = 7.5 V

**Version 3:** *ISRC* = 2 A, *vS*(*t*) = 5*ISRC* = 10 V

**Version 4:** *ISRC* = 2.5 A, *vS*(*t*) = 5*ISRC* = 12.5 V

**Version 5:** *ISRC* = 3 A, *vS*(*t*) = 5*ISRC* = 15 V

3%

**11.** The switch is closed at *t* = 0 with no energy initially stored in the circuit. Determine the largest value of *di*/*dt*, assuming *VSRC* = 2 V.

**Solution:** The largest value of *di*/*dt* occurs at *t* = 0, when *vL* = *Ldi*/*dt* = *VSRC*, and *di*/*dt* = *VSRC*,/*L* = 103*VSRC* A/s = *VSRC* A/ms, where *VSRC* is in V.

**Version 1:** *VSRC* = 2 V, *dit/dt* = *VSRC* = 2 A/ms

**Version 2:** *VSRC* = 3 V, *dit/dt* = *VSRC* = 3 A/ms

**Version 3:** *VSRC* = 4 V, *dit/dt* = *VSRC* = 4 A/ms

**Version 4:** *VSRC* = 5 V, *dit/dt* = *VSRC* = 5 A/ms

**Version 5:** *VSRC* = 6 V, *dit/dt* = *VSRC* = 6 A/ms

3%

**12.** When the switch is opened at *t* = 0, with no energy initially stored in the circuit, it is found that *v*(*t*) = 2*t* V, *t* ≥ 0+ *t* is in s. Determine *ISRC.*

**Solution:** At *t* = 0+, *ISRC* flows in *C*; *iC*(*t*) = *Cdv*/*dt* = *Cd*(*Kt*)/*dt* = *CK* = *CK* at *t* = 0+. It follows that *ISRC = CK* = 0.5*K*.

**Version 1:** *K* = 2 V/s, *ISRC* = 1 A

**Version 2:** *K* = 3 V/s, *ISRC* = 1.5 A

**Version 3:** *K* = 4 V/s, *ISRC* = 2 A

**Version 4:** *K* = 5 V/s, *ISRC* = 2.5 A

**Version 5:** *K* = 6 V/s, *ISRC* = 3 A

13%

**13.** Determine the average power dissipated in the load, assuming *vSRC*(*t*) = 10cos(*ωt* + 45°) V and *R* = 2 Ω.

**Solution:** **VSRC** = 10∠45° V. Let **V** be the voltage across the primary winding, **I** be the load current, and **IS** be the source current, as shown. It follows that **V** = *ZL***I**, and **V** = **VSRC** – *ZS***IS** (1), where *ZL* = (*R* + *j*10) Ω, and *ZL* = (4*R* – *j*10) Ω. From the mmf equation:

*N***IS** + *N***I** – 2*N***I** = 0, which gives **IS** = **I**. Substituting for **V** and **IS** in (1) and solving for **I**:  A. *P* =  W. Note that in this case, because the same current flows in the two secondary windings, the transformer can be considered as a 1:1 ideal transformer without voltage inversion; that is, the voltage across the primary winding is the load voltage, and the current in the primary winding is the load current. It follows that , as before.

**Version 1:** *R* = 2 Ω; *P* = 1 W

**Version 2:** *R* = 4 Ω; *P* = 0.5 W

**Version 3:** *R* = 8 Ω; *P* = 0.25 W

**Version 4:** *R* = 10 Ω; *P* = 0.2 W

**Version 5:** *R* = 20 Ω; *P* = 0.1 W

13%

**14.** Derive TEC looking into terminals ‘ab’, where **VTH** = **Vab** and **ISRC** = 0.1∠0° A.

**Solution:** Replacing the two coupled coils by the T-equivalent circuit, the circuit becomes as shown. 100**I** – 200**I** = 0, so that **I** = 0. It follows that **Va** = *j*30**ISRC**, and **V1** = -*j*50**ISRC**. Hence, **Vb** = -2**V1** = *j*100**ISRC**. This gives **VTh** = *j*30**ISRC** – *j*100**ISRC** = -*j*70**ISRC**.

To determine *ZTh*, a test current source **IT** is applied, with the source **ISRC** replaced by an open circuit. The circuit becomes as shown. 100**I** – 200(**I** – **IT**) = 0, so that **I** = 2**IT**. From KVL around the outer loop, **V1** = *j*30(**I** – **IT**) + *j*20**I** = *j*70**IT**. From KVL around the upper mesh, **Vab** =

 (-*j*10)**IT** + *j*20**I** + **V1**. Substituting for **I** and **V1** in terms of **IT**, **Vab** = *j*30**IT** + *j*70**IT** = *j*100**IT**. this gives **Vab/IT** =

*ZTh* = *j*100 Ω.

**Version 1:** **ISRC** = 0.1∠0° A; **VTh** = -*j*70**ISRC** = 7∠-90° V

**Version 2:** **ISRC** = 0.2∠0° A; **VTh** = -*j*70**ISRC** = 14∠-90° V

**Version 3:** **ISRC** = 0.3∠0° A; **VTh** = -*j*70**ISRC** = 21∠-90° V

**Version 4:** **ISRC** = 0.4∠0° A; **VTh** = -*j*70**ISRC** = 28∠-90° V

**Version 5:** **ISRC** = 0.5∠0° A; **VTh** = -*j*70**ISRC** = 35∠-90° V

13%

**15.** Switch S1 is closed at *t* = 0, with the capacitor initially uncharged. Switch S2 is closed at *t* = 30 ms, and switch S3 is closed at *t* = 50 ms. Determine *vC*(*t*) for all

*t* ≥ 0+.

**Solution:** *vC*(0+) = 0, *VCF*1 *=* 2×10-3×20×103 = 40 V; *τ*1 = 20×103×0.5×10-6 ≡ 10 ms. Hence,

*vC*(*t*) =  V, 0 ≤ *t* ≤ 30 ms. At *t* = 30 ms, *vC*(30ms) =  = 38.0 V; the circuit seen by the capacitor for 30 ms ≤ *t* ≤ 50 ms is as shown. The 15 V source in series with 5 kΩ is transformed to a 3 mA current source in parallel with 5 kΩ. The two current sources are combined into a 5 mA source and the parallel resistance is (20×5)/25 = 4 kΩ; hence *VCF*2 *=* 20 V. *τ*2 = 4×0.5 = 2 ms. Closing S3 at *t* = 50 ms has no effect on *vC*. It follows that *vC*(*t*) = V, or *vC*(*t*) =  V, *t* ≥ 30 ms. Closing S3 places the 20 kΩ resistor across an ideal voltage source, which does not affect *vC*.

13%

**16.** The switch is opened at *t* = 0 after being closed for a long time: (a) determine *R* for critical damping (4 grades); using this value of *R*, and assuming *VSRC* = 6 V, determine: (b) *vC*(*t*), *t* ≥ 0+, *t* in ms; (6 grades); (c) the energy in μJ stored in the capacitor and inductor as functions of time, *t* ≥ 0+, *t* in ms (3 grades).

**Solution:** (a)  rad/s. , *R* = 0.1×1000 = 100 Ω.

(b) *vC*(*t*) = ; at *t* = 0+, *vC*(0+) = *VSRC* = *A*; ; *iL*(0+) = , , which gives *B* = 0. Hence, , *t* ≥ 0+, *t* is in s, or, , *t* ≥ 0+, *t* is in ms. The response is that of a first-order circuit.

(c) Energy stored in *C*:  μJ*, t* ≥ 0+, *t* is in ms, , , *t* is in s; energy stored in *L*: , , *t* ≥ 0+, *t* is in ms. The energy stored in the capacitor and inductor is the same. The inductor current is in a direction to discharge the capacitor. Hence, the two stored energies decay together, like a single stored energy. This is the fundamental reason behind the first-order response.

**Version 1:** *VSRC* = 2 V, = 20

**Version 2:** *VSRC* = 3 V, = 45

**Version 3:** *VSRC* = 4 V, = 80

**Version 4:** *VSRC* = 5 V, = 125

**Version 5:** *VSRC* = 6 V, = 180